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A DYNAMIC MODEL OF A THERMOELASTIC CONTINUOUS MEDIUM

WITH PRESSURE RELAXATION

A. M. Iskol'dskii and E. I. Romenskii                          UDC 531;621.316

The dynamics of the pulsed heating of a metal to submelting temperatures by an electric current was analyzed in [1] on the basis of the Maxwellian model of a nonlinearly elastic medium with relaxation of shear stresses [2]. A number of experimental relationships which seemed anomalous within the framework of simplified models were explained in this case. In [1] it was assumed that bulk deformations take place elastically.

In the present work a model with relaxation of bulk deformations in a liquid (shear stresses are ignored) is formulated which allows one to give a natural interpretation of an electrical explosion as a phenomenon arising when the level of specific energy content of the medium is inexplicably high. In particular, when a copper conductor is heated in an electric circuit providing a rate of temperature rise $dT/dt \approx 1.5 \cdot 10^{10}$ deg/sec, the starting point of the electrical explosion comes at an energy $Q_* \approx 3.2$ kJ/g (T $\approx$ 6000°K), whereas under equilibrium conditions (p = 1013 hPa) boiling starts at $Q_{boil} \approx 1.35$ kJ/g (T $\approx$ 2900°K).

Such a result cannot be explained within the framework of the theory of metastable states (the Zel'dovich–Fol'mer theory of nucleation, in particular; also see [3]) if plausible estimates are used for the work of formation of the critical nucleus and for the value of the preexponent in the universal expression for the flux of nuclei in the region of sizes larger than the critical size.

Allowance for bulk relaxation also seems a necessary expansion of the model with shear relaxation for the region of lower (submelting) temperatures.

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1. Let us consider a continuous medium in which deformations are accompanied only by variation of volume. In this case, the stress tensor is described by a scalar — the pressure. In the state of thermodynamic equilibrium such a medium is described by two thermodynamic parameters, such as the density $\rho$ and the entropy S.

Now let us assume that an irreversible change in volume, connected with the creation and disappearance of defects, can occur in the medium. We introduce the value $\rho_*$ of the density which an element of the medium takes after the establishment of thermodynamic equilibrium, it having a density $\rho$ at the given time. We designate the characteristic time of establishment of equilibrium as $\tau$. Thus, a new thermodynamic variable $\rho_*$ is introduced, and, in accordance with this, we have for the internal energy the identity

$$dE = -p\,dv + T\,dS + q\,dv_* ,$$

(1.1)

where $v = 1/\rho$ is the specific volume; $v_* = 1/\rho_*$ is the specific volume in the equilibrium state; $p = -E_v$ is the pressure; $T = E_S$ is the temperature.

The equations of dynamics of such a medium follow from the laws of conservation of mass, momentum, and energy, and are also supplemented by the relaxation equation

$$\rho\,du_i/dt + \partial p/\partial x_i = 0, \quad \rho\,dE/dt + p(\partial u_1/\partial x_1 + \partial u_2/\partial x_2 + \partial u_3/\partial x_3) = 0,$$

$$d\rho/dt + \rho\,(\partial u_1/\partial x_1 + \partial u_2/\partial x_2 + \partial u_3/\partial x_3) = 0, \quad d\rho_*/dt = -\rho_* \ln(\rho_*/\rho)/\tau,$$

(1.2)

where $u_i$ is the velocity vector; $d/dt = \partial/\partial t + u_\alpha \partial/\partial x_\alpha$ is the derivative along the trajectory of motion.

Relaxation of the density to its equilibrium value is postulated in the last equation of the system (1.2).

We note that for the parameter $\xi = \ln(\rho_*/\rho)$ we get from (1.2) the relaxation equation

$$d\xi/dt = \partial u_1/\partial x_1 + \partial u_2/\partial x_2 + \partial u_3/\partial x_3 - \xi/\tau.$$

Now $\xi$ rather than $v_*$ characterizes the departure of the medium from thermodynamic equilibrium: $E = E(\rho, S, \xi)$.

From (1.2) we get the equation of evolution of entropy

$$\frac{dS}{dt} = \frac{\xi E_\xi}{(E_S \tau)}.$$

Thus, entropy is not conserved here, and the process becomes irreversible. The demand that the entropy not decrease imposes restrictions on the dependence of E on $\xi$ in the form of the inequality ($E_S = T > 0$, $\tau > 0$)

$$\xi E_\xi \geqslant 0.$$

(1.3)

The dependence of E on $\xi$ must be chosen in such a way as to describe in some measure the experimental facts, which will be discussed below. As the simplest such dependence we take

$$E(\rho, S, \xi) = E^0(\rho, S) + \frac{1}{2} M(\rho, S) \xi^2.$$

(1.4)

It is obvious that (1.4) satisfies (1.3) if $M(\rho, S) > 0$, and this will be presumed from now on. The pressure and temperature calculated from (1.4) have the form

$$p = \rho^2 E_\rho = \rho^2 E_\rho^0 + \frac{1}{2}\rho^2 M_\rho \xi^2 - \rho M \xi, \quad T = E_S = E_S^0 + \frac{1}{2} M_S \xi^2.$$

(1.5)

The behavior of metals during pulsed heating is discussed primarily in the present article. The model under consideration corresponds to the physical situation in which the characteristic time of formation of thermodynamic equilibrium of the atomic configuration, connected with the dynamics (creation and mutual orientation) of defects, is considerably longer than the phonon time. A nonequilibrium "intermediate" field of deformations, characterized by the new thermodynamic coordinate $\xi$, arises in this case. In the absence of outside actions the $\xi$ field relaxes, and the energy stored in it is redistributed over the equilibrium degrees of freedom. This is a model of a continuous medium if each element of it contains a significant number of defects while the size of an element exceeds the distance between defects.

The model is intended, in particular, for the description of metals in the region of submelting temperatures. In this region the main type of defects in thermodynamic equilibrium is vacancies, which occupy up to several percent of the specific volume. The degree of defectiveness of a liquid is still higher.

2. To close the model we must determine the internal energy $E(\rho, S, \xi)$ and the relaxation time $\tau$. Let us consider several examples allowing us to obtain preliminary information about the modulus M in Eq. (1.4) and about the time $\tau$, which can be a function of the state of the medium.

Let us consider the problem of the pulsed heating of an element of the medium. We assume that the element is small enough that the gradients of all the parameters describing the state of the medium can be neglected. Suppose the outside pressure equals zero while the temperature within the element varies by an assigned law. This process can be described by the equations [we assume that the modulus M in (1.4) does not depend on $\rho$ or S]

$$T(\rho, S) = T(t), \quad p(\rho, S, \xi) = 0,$$
$$d \ln (\rho_*/\rho_0)/dt = -\ln (\rho_*/\rho)/\tau. \tag{2.1}$$

Here $\rho_0$ is the initial density of the medium at $t = 0$; $T(t)$ is the assigned function; $p = \rho^2 E_\rho(\rho, S, \rho_*)$ is the pressure determined from (1.5).

Differentiating the first two equations of (2.1) with respect to t, using the relation

$$\frac{\rho_*}{\rho} p_{\rho_*} = -M,$$

which follows from (1.5) with M = const, we write the system (2.1) in the form

$$\frac{d \ln \rho/\rho_0}{dt} = -\alpha \dot{T} - \frac{M}{K} \frac{\xi}{\tau}; \tag{2.2}$$

$$\frac{d\xi}{dt} = \alpha \dot{T} - \frac{K - M}{K} \frac{\xi}{\tau}; \tag{2.3}$$

$$\frac{dS}{dt} = \frac{c_p}{T} \dot{T} + \alpha M \frac{\xi}{\tau}, \tag{2.4}$$

where $c_p$, $\alpha$, and K are the heat capacity at constant pressure, the volumetric expansion coefficient, and the isothermal bulk modulus, respectively, calculated using (1.1) with $\rho_* =$ const, i.e., in a medium with an equilibrium concentration of defects. In a first approximation they can be considered as corresponding to a defect-free medium.

Below we shall take $\dot{T} =$ const and $\tau =$ const. First we consider the case of "superfast" heating of the element, assuming that the heating time is $t_h \ll \tau$. We designate $T_{max} = \dot{T} t_h$. From Eqs. (2.2) and (2.3), with $t = t_h$, we obtain

$$\xi_{max} = \alpha T_{max}, \quad \ln \rho_{max}/\rho_0 = -\alpha T_{max},$$

from which $\rho_* |_{t=t_h} = \rho_0$, i.e., fast heating does not result in the creation of defects.

Now we stop the heating instantaneously, taking $\dot{T} = 0$, and follow the relaxation process — the process of establishment of the equilibrium concentration of defects.

From (2.2)-(2.4) we get

$$\xi = \xi_{max} \exp\left(\frac{K - M}{K} \frac{t_h - t}{\tau}\right),$$

$$\ln (\rho/\rho_0) = \ln (\rho_{max}/\rho_0) - \xi_{max} \frac{M}{K - M} \left(1 - \exp\left(\frac{K - M}{K} \frac{t_h - t}{\tau}\right)\right),$$

$$S = S|_{t=t_h} + \xi_{max} \frac{\alpha M K}{K - M} \left(1 - \exp\left(\frac{K - M}{K} \frac{t_h - t}{\tau}\right)\right).$$

From this we can calculate the volumetric expansion coefficient $\alpha'$ and the heat capacity $c'_p$ in the process of relaxation. Varying $T_{max}$ through variation of the heating rate, $\delta T_{max} = (\delta \dot{T}) t_h$, we obtain

$$\alpha' = -\frac{\delta \ln(\rho/\rho_0)}{\delta T_{max}} = \alpha + \frac{\alpha M}{K-M}\left(1 - \frac{\xi}{\xi_{max}}\right); \tag{2.5}$$

$$c_p' = T_{max}\frac{\delta S}{\delta T_{max}} = c_p + \frac{\alpha^2 MKT_{max}}{K-M}\left(1 - \frac{\xi}{\xi_{max}}\right). \tag{2.6}$$

Thus, expressions are obtained for the corrections to the volumetric expansion coefficient $\alpha$ and the heat capacity $c_p$ connected with the formation of the equilibrium concentration of defects during relaxation. It follows from (2.5) and (2.6) that some caution must be displayed in interpreting the results of dynamic measurements of that fraction of the thermal expansion and heat capacity which is controlled by defects: In static measurements $\xi = 0$, while in dynamic ones a nonzero level of nonequilibrium is established.

Now let us consider the process of heating at a rate of temperature rise $\dot{T}$.

We assume that in a certain interval of variation of the thermodynamic parameters, varies little as a function of the state of the medium. In this case, the situation when $d\xi/dt$ in Eq. (2.3) is a small quantity is possible, and then

$$\xi = \tau\dot{T}\alpha K/(K-M). \tag{2.7}$$

For the density and entropy in this approximation we obtain from (2.2) and (2.4)

$$\frac{d\ln(\rho/\rho_0)}{dt} = -\alpha\dot{T}\left(1 + \frac{M}{K-M}\right), \quad \frac{dS}{dt} = \frac{c_p}{T}\dot{T}\left(1 + \frac{\alpha^2 TMK}{c_p(K-M)}\right).$$

Eliminating dt from these equations, we have

$$\frac{1}{v}\frac{dv}{dT} = \alpha + \frac{\alpha M}{K-M} = \alpha + \Delta\alpha, \quad T\frac{dS}{dT} = c_p + \frac{\alpha^2 MKT}{K-M} = c_p + \Delta c_p.$$

Thus, the corrections to the volumetric thermal expansion coefficient and the heat capacity are obtained for an established process ($d\xi/dt = 0$):

$$\Delta\alpha = \alpha M/(K-M); \tag{2.8}$$

$$\Delta c_p = \frac{\alpha^2 MKT}{K-M} = (c_p - c_v)\frac{M}{K-M}. \tag{2.9}$$

We note that in the given case a nonequilibrium pressure associated with the $\xi$ field is formed: From (2.7) and (1.5) we get

$$p = p^0(\rho, S) - \rho\tau\dot{T}\alpha MK/(K-M). \tag{2.10}$$

The second term in this expression is connected with the effective volumetric viscosity.

Equations (2.8) and (2.9) for the corrections to the volumetric expansion coefficient $\alpha$ and the heat capacity $c_p$ under the conditions $p = const$ and $\xi = const$ can be obtained from Eqs. (1.5) for p and T with M = const.

Thus, the problem of pulsed heating in the approximation under consideration makes it possible to obtain a description of such characteristics of the medium as the thermal expansion coefficient, the heat capacity at constant pressure, and the nonequilibrium pressure. These characteristics can be used to close the model. We present certain experimental data allowing one to estimate the intervals of variation of the parameters (M and $\tau$, in particular) needed to close the model. The figures obtained below can be refined when necessary by choosing reasonable functional relations $M(\rho, S)$ and $\tau(\rho, S)$.

To estimate the modulus M for metals in the submelting region one can use data of precise measurements, of the temperature dependence of the total thermal expansion, and of that fraction of it which is connected only with the increase in the size of the lattice [see Eq. (2.5)] or of the analogous relation for the heat capacity of vacancies [see Eq. (2.6)], for example.

In the heating of refractory metals to the melting temperature about half the increment of the thermal expansion coefficient is controlled by defects [4, 5]. There are a number of methods of determining the required relations. In particular, x-ray measurements allow one to determine directly the characteristic size of the lattice [6]. According to the available data for metals in the solid phase, $M/(K-M) = (c^2_\infty - c^2_0)/c^2_0 < 1$ (definitions of $c_\infty$ and $c_0$
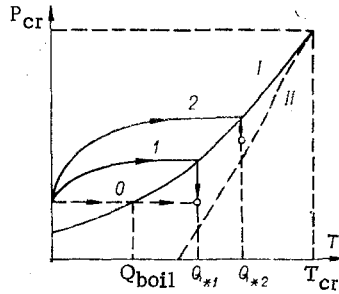
Fig. 1

will be given below). According to the data of various methods for copper in the solid phase near the melting temperature, for example, $\Delta\alpha/\alpha = M/(K - M) = 0.1$-$0.2$.

The relaxation time $\tau$ is controlled by a whole series of relaxation processes with the participation of defects in thermodynamic equilibrium [7]. This time can be considered as the "lifetime" of a defect. The quantity $\tau$ depends essentially on the temperature and can be found, for example, from experiments on the hardening of vacancies (in the given model hardening occurs if the characteristic time of cooling of the specimen is less than the relaxation time). For an estimate we can set $\tau \gg 10^{-6}$ sec, since, according to experimental data, a cooling rate of about $10^6$ deg/sec is sufficient for the density of vacancies at the end of the process to differ insignificantly from the initial density.

Taking $\tau \sim 10^{-5}$ sec and $M \sim 2 \cdot 10^{10}$ dyn/cm$^2$ for copper in the submelting region, we have an estimate of $\eta \simeq 10^6$ P for the effective volumetric viscosity.

The nonequilibrium pressure [see (2.10)], developing in the process of heating of the liquid, results in a shift of the phase trajectory of the heated element into the region of stable states (see Fig. 1), lying above the curve of the liquid–vapor equilibrium phase transition on the p–T diagram. After $\xi$ arrives at the stationary level (2.7), this point moves toward the right almost parallel to the temperature axis, owing to the continuing heating, until it meets the equilibrium curve. Further heating results in boiling. This aspect is well recorded experimentally: A sharp increase in the rate of growth of the transverse size of the conductor is observed in x-ray shadow photographs; from the surface of the conductor emerge streams of high-temperature dense gas which, having overtaken the air–vapor boundary which has gone ahead, are revealed in the form of bright regions on photographs made with streak cameras and image-converter cameras; electrical measurements show that the process of effective interruption of the current starts at this instant.

Phase trajectories in the p–T plane of the point corresponding to the surface of the conductor are presented in Fig. 1: I is the line of the equilibrium liquid–vapor phase transition; II is the spinodal; ($p_{cr}$, $T_{cr}$) are the coordinates of the critical point; $Q_{boil}$ is the point of the onset of boiling at atmospheric pressure; $Q_{*1}$ and $Q_{*2}$ are the starting points of the electrical explosion; 0 is an unrealized trajectory; 1 is the true trajectory for the same heating rate as for the case 0; 2 is the same, but at a higher heating rate.

Knowing the equation for the equilibrium curve and the position of the starting point of the electrical explosion on the temperature (energy) axis, one can determine the amplitude of the nonequilibrium pressure and, on the basis of (2.10), estimate the effective volumetric viscosity of the liquid metal at temperatures exceeding the equilibrium boiling temperature. Far from the critical point the equation for the curve of the equilibrium liquid–vapor phase transition has the form

$$p = p_\infty e^{-\lambda/RT}, \tag{2.11}$$

where $\lambda$ is the heat of the transition while $p_\infty$ is determined from the known saturated vapor pressure at a fixed temperature. For copper at $T = 2900°K$, in particular, $p = 1013$ hPa, $T_\infty = \lambda/R = 36.56 \cdot 10^3 °K$, and $p_\infty \simeq 3.35$ GPa.

According to our experimental data for copper at a heating rate $dT/dt \simeq (1.5$-$1.6) \cdot 10^{10}$ deg/sec, the phase trajectory intersects the line of the equilibrium phase transition when the average (radially) value of the internal energy density is $Q_* = 3.1 \pm 0.1$ kJ/g. Experimental conditions: conductor diameter 0.4 mm, current at the instant of the explosion $I_* \simeq 50$ kA, voltage on the specimen 13.3 kV (storage capacitance 2.4 µF, circuit inductance 430

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nH, initial voltage on capacitance 30 kV). The density of the medium at this instant, estimated from x-ray shadow photographs, is $n_* = 5.8 \pm 0.2$ g/cm$^3$. The volumetric thermal expansion coefficient found from the same photographs and the data of electrical measurements is $\alpha = (1.4 \pm 0.1) \cdot 10^{-4}$ deg$^{-1}$. In the conversion from internal energy density to temperature we take $c = 0.5$ J/(g·deg) as the heat capacity of a unit mass. The singular point falls at the temperature $T_* = 6.44 \cdot 10^3$ °K (actually, the surface temperature is lower [1]).

On the basis of (2.11) we obtain the pressure estimate

$$p_* = p_\infty \, e^{-T_\infty/T_*} \cong 0.135 \, \text{GPa},$$

which, in accordance with the foregoing, should be close to (2.10):

$$p = \rho \tau \frac{dT}{dt} \frac{\alpha M K}{K - M}.$$

Hence, $\tau M K/(K - M) \simeq 100$ cm$^2$/sec, and for the effective volumetric kinematic viscosity $\nu = \tau M$ (we obtain an expression for it below) we obtain $\nu = 5$–$100$ cm$^2$/sec, while for the dynamic viscosity $\eta$ we have $\eta = 300$–$600$ P, respectively (here the following limits of variation of M are presumed: from $K \simeq K - M$ to $M \simeq K - M$).

From $p = p_*$ we get the experimentally verifiable relation (we presume $K \simeq K - M$)

$$T \cong T_\infty / \ln \, (p_\infty / \rho \tau \alpha M \dot{T}), \tag{2.12}$$

connecting the position of the starting point of the electrical explosion on the energy axis with the heating rate. According to (2.12), under the conditions of [8], where the explosion of a conductor 0.31 mm in diameter at a heating rate about five times lower than that just discussed was investigated, one can expect a shift of the starting point of the explosion into the energy region close to $Q_* = 2.4$ kJ/g $= 152$ kJ/mole. According to the data of [8], it falls at $\hat{Q}_* \simeq 152$ kJ/mole. The agreement in the third figure is accidental, of course. However, the estimates for $\nu \simeq 50$ cm$^2$/sec and $\eta = 300$ P are close to the real ones in order of magnitude.

Since the degree of defectiveness of a material increases during melting, it can be expected that $M \simeq K - M$ in the liquid phase. Taking $K \sim 10^{11}$ cm$^2$/sec$^2$, we obtain $\tau \sim 5 \cdot 10^{-10}$ sec, while the characteristic heating time $[(1/T)(dT/dt)]^{-1}$ is considerably longer than $\tau$ in the cases under consideration ($\omega\tau \ll 1$).

However, $\omega\tau \gtrsim 1$ in explosions of emission spots on the cathodes of high-current commutators, in the interaction of beams of energetic particles with the surfaces of metallic targets, in experiments with powerful laser beams, etc.

Under these conditions the transition to a model in local equilibrium with a renormalized volumetric viscosity coefficient proves incorrect, and the relaxation must be taken into account as such.

3. Liquids with relaxing parameters have been analyzed repeatedly. We note [9], for example, where the propagation of acoustic waves is investigated, in particular.

We can show that the model formulated above leads to the same results as in [9]. We assume that a small disturbance $\delta\rho$, $\delta\rho_*$, $\delta S$, $\delta u_i$ is imposed on the steady state of the medium $\rho^0 = $ const, $\rho^0_* = \rho^0$, $S^0 = $ const, $u^0_i = $ const. We linearize the system (1.2) with respect to the small disturbances. We obtain the system of linear differential equations

$$\rho^0 d\delta u_i/dt + \partial \delta p/\partial x_i = 0, \quad d\delta\rho/dt + \rho^0(\partial \delta u_1/\partial x_1 + \partial \delta u_2/\partial x_2 + \partial \delta u_3/\partial x_3) = 0, \tag{3.1}$$

$$d\delta\rho_*/dt = -(\delta\rho_* - \delta\rho)/\tau^0, \quad d\delta S/dt = 0,$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u^0_\alpha \frac{\partial}{\partial x_\alpha}; \quad \delta p = \left(\frac{\partial p}{\partial \rho}\right)_{\xi,S} \delta\rho + \left(\frac{\partial p}{\partial S}\right)_{\xi,\rho} \delta S + \left(\frac{\partial p}{\partial \xi}\right)_{\rho,S} \delta\xi.$$

For $\delta\xi$ we obtain the equation

$$d\delta\xi/dt = \partial \delta u_1/\partial x_1 + \partial \delta u_2/\partial x_2 + \partial \delta u_3/\partial x_3 - \delta\xi/\tau^0.$$

We initially assume that $u^0{}_\alpha = 0$. Let the disturbances vary with time by the harmonic law $e^{i\omega t}$. From the equation for $\delta\xi$ we find

$$\delta\xi(x_1, x_2, x_3) = -\frac{i\omega\tau^0}{1 + i\omega\tau^0}\frac{\delta\rho}{\rho^0},$$

and hence, because $\delta S(x_1, x_2, x_3) = 0$,

$$\delta p(x_1, x_2, x_3) = \left(\left(\frac{\partial p}{\partial\rho}\right)_{\xi,S} - \frac{1}{\rho^0}\left(\frac{\partial p}{\partial\xi}\right)_{\rho,S}\frac{i\omega\tau^0}{1 + i\omega\tau^0}\right)\delta p(x_1, x_2, x_3). \tag{3.2}$$

Now we seek solutions of the type $e^{ik_\alpha x_\alpha}$.

From the condition of solvability of the system (3.1) with $u^0{}_\alpha = 0$ we obtain the dispersion relation

$$\omega^2 = \left(\left(\frac{\partial p}{\partial\rho}\right)_{\xi,S} - \frac{1}{\rho^0}\left(\frac{\partial p}{\partial\xi}\right)_{\rho,S}\frac{i\omega\tau^0}{1 + i\omega\tau^0}\right)\left(k_1^2 + k_2^2 + k_3^2\right). \tag{3.3}$$

One can show that the same equation (3.3) in which $\omega$ is replaced by $\omega + u^0{}_\alpha k_\alpha$ is valid for the case of $u^0{}_\alpha \neq 0$.

Thus, an expression is obtained for the complex speed of sound (3.3) coinciding with the equation of [9].

Now let us consider the limiting cases for (3.3). We assume that the relaxation time $\tau$ can vary within wide limits, and we fix the frequency $\omega$. Let $\tau \gg 1/\omega$, i.e., the relaxation time is long and the medium is "almost elastic." From (3.3) we get

$$\omega^2 = \left(\left(\frac{\partial p}{\partial\rho}\right)_{\xi,S} - \frac{1}{\rho^0}\left(\frac{\partial p}{\partial\xi}\right)_{\rho,S}\right)|k|^2,$$

where $|k|^2 = k_1^2 + k_2^2 + k_3^2$. The speed of sound $c_\infty$, defined as

$$c_\infty^2 = \left(\frac{\partial p}{\partial\rho}\right)_{\xi,S} - \frac{1}{\rho^0}\left(\frac{\partial p}{\partial\xi}\right)_{\rho,S},$$

corresponds to the speed of sound in a medium without relaxation.

For $\tau \ll 1/\omega$ (a short relaxation time) we have

$$\omega^2 = (\partial p/\partial\rho)_{\xi,S}|k|^2.$$

The speed of sound $c_0 = \sqrt{(\partial p/\partial\rho)_{\xi,S}}$ corresponds to an "instantly relaxing" medium.

Consequently,

$$(\partial p/\partial\xi)_{\rho,S} = -\rho^0\left(c_\infty^2 - c_0^2\right).$$

For the case of short relaxation times ($\tau \ll 1/\omega$), using $i\omega\delta\rho + \rho^0\,\text{div}\,\delta u = 0$, we obtain from (3.2)

$$\delta p = \left(\frac{\partial p}{\partial\rho}\right)_{\xi,S}\delta\rho - \left(c_\infty^2 - c_0^2\right)\tau^0\,\text{div}\,\delta u. \tag{3.4}$$

An equation containing the volumetric viscosity has been obtained for the pressure.

The effective volumetric kinematic viscosity is expressed by the equation [using (1.5) with M = const],

$$v = \tau^0\left(c_\infty^2 - c_0^2\right) = -\frac{\tau^0}{\rho^0}\left(\frac{\partial p}{\partial\xi}\right)_{\rho,S} = \tau^0 M. \tag{3.5}$$

We note that Eq. (3.4) for the pressure containing the volumetric viscosity (3.5) is valid only for processes sufficiently slow compared to the relaxation time $\tau$. Consequently, the model with Navier–Stokes viscosity determined from (3.5) is inapplicable for rapidly occurring processes.

Also significant is the fact that the system of equations (1.2) with density relaxation is hyperbolic (all the processes have a finite propagation rate).

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# EQUATIONS OF DISLOCATION PLASTICITY WITH LARGE DEFORMATIONS

Yu. I. Fadeenko

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In investigations of the dynamic plasticity the simplest model of the theory of dislocation, including Orovan's equation

$$\dot{\gamma} = bNv, \tag{1a}$$

the law governing the motion of the dislocations

$$v = v(\tau) \tag{1b}$$

and the equation of kinetics of dislocations

$$N = N(\gamma), \tag{1c}$$

where $\gamma$ is the shear deformation; $\tau$, tangential stress; $N$, density of dislocations; $v$, their slipping velocity; and $b$, absolute magnitude of Burgers vector, is often used. It is also assumed that all dislocations are mobile and slip with identical velocities. Equation (1c) is usually written in the form $N = N_0 + A\gamma^s$, where s is a quantity of the order of one, and the motion is described either by the law of viscous friction

$$\tau b = Bv, \tag{2a}$$

or by Taylor's empirical formula [1]

$$v = v_0 \exp(-\tau_0/\tau) \tag{2b}$$

(B is the coefficient of viscous friction).

This very simple model corresponds to conditions of superbarrier slipping with uniform chaotic distribution of dislocations and can be used to describe small deformations of metals with low initial dislocation density. However, it neglects the fact that the uniform dis-